43. Find three positive numbers whose sum is 100 and whose product is a maximum.

$xyz \to \max.$

$x+y+z=100$

$\nabla f = \langle yz, xz, xy \rangle$

$\nabla h = \langle 1, 1, 1 \rangle$

$yz = \lambda(1) \qquad xy = \lambda(1)$

$xz = \lambda(1)$

$\frac{yz}{z} = \frac{xz}{z} = \frac{xy}{z}$ | $\frac{yz}{x} = \frac{xz}{x} = \frac{xy}{x}$

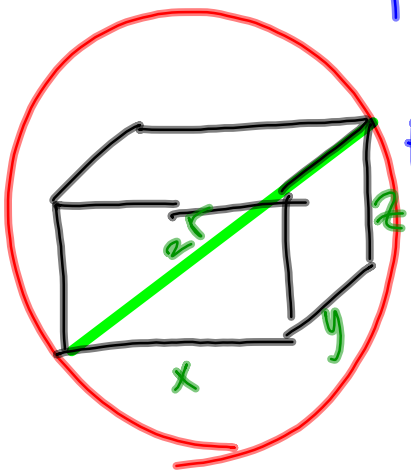
$y = x = \frac{xy}{z}$ | $\frac{yz}{x} = z = y$

$x+y+z=100$

$y+y+y=100$

$y = 33.\overline{3}$

45. Find the maximum volume of a rectangular box that is inscribed in a sphere of radius r.



$F_1 = x^2 + y^2 + z^2 = 4r^2$

$f_2 = V = xyz$

$\nabla F_1 = \langle 2x, 2y, 2z \rangle$

$\nabla F_2 = \langle yz, xz, xy \rangle$

$yz = 2x(\lambda)$

$xz = 2y(\lambda)$

$xy = 2z(\lambda)$

$\frac{yz}{2x} = \frac{xz}{2y} \Rightarrow y^2 = x^2 = z^2$

$\frac{xz}{2y} = \frac{xy}{2z} \Rightarrow z^2 = y^2$

$3z^2 = 4r^2$

$z^2 = \frac{4}{3}r^2$

$\therefore V = r^3 = \left(\frac{4}{3}\right)^{\frac{3}{2}} r^3$

41. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin.

$f = d^2 = x^2 + y^2 + z^2$